Creep Prediction Using The
Non-Linear Strain Energy Equivalence Theory

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Abstract

The Non-Linear Strain Energy Equivalence Theory, a semi-empirical model, is utilized to predict long-term creep from short-term compressive stress-strain experiments conducted at different strain rates. Stress-strain experiments in uni-axial compression are performed at strain rates of 3 and 0.03 %/minute to predict creep behavior and stress-strain data at several strain rates for an immiscible polymer blend of recycled fractional melt flow high-density polyethylene and recycled polystyrene. The creep behavior is predicted up to 50 years at stress levels of 400 and 800 psi.

Introduction

The Non-Linear Strain Energy Equivalence Theory is a correlative method in which data from two short-term stress-strain experiments conducted at different strain rates, each corresponding to a specific length of time, are used to predict long-term creep strain, the creep exponent, and stress-strain data at any strain rate. Only two short-term stress-strain experiments are required to predict the creep behavior at as many stress levels as desired from the same two experiments for any material in any structural loading application with known creep loading conditions. Prediction of stress-strain data at any strain rate allows retrieval of the entire stress-strain curve, for modulus, yield stress, ultimate stress, and toughness determination

The theory has been tested against several types of polymeric based materials with a wide range of morphologies and mechanical properties.[1] Single-phase systems (HDPE), two-phase polymer systems (PS/HDPE and PP/HDPE), and multi-component systems (glass fiber reinforced HDPE) were studied for times up to 26 years.

In this work, the creep behavior of an immiscible polymer blend of 35% PS and 65% HDPE is predicted for times up to 50 years at stress levels of 400 and 800 psi from short-term compressive stress-strain experimental data.

Experimental Stress-Strain Procedure

Uniaxial compressive stress-strain experiments were conducted at two constant strain rates using a MTS Model 810 servo-hydraulic test machine. A strain rate of 3 %/min was chosen satisfying ASTM D6108. In addition, a rate of 0.03 %/minute, 100 times slower than the 3 %/min rate, was chosen and functions as the reference strain rate. The strain rates are referred to as $\dot{\varepsilon}_1 = 0.0003$ /min and $\dot{\varepsilon}_2 = 0.03$ /min.

Test samples were cut from five different molded profiles with a four by four inch cross-section and a twelve-foot length. The five profiles, utilized to demonstrate experimental repeatability and quality control, were cut into test specimens of 7.25 inches in length, or twice the minimum cross sectional dimension, as according to ASTM D 6108. The cut surfaces of the test specimens were then machined to assure smooth and parallel end surfaces. The density, specific gravity, and cross sectional area were determined for each sample by the water displacement method, ASTM D 6111, prior to performing the compressive property testing.

Each sample was preloaded to approximately 80 lbs to assure proper contact, stability, and alignment of the test specimen and the parallel testing plates and to minimize the toe correction. The results of five tested samples were averaged for each strain rate.

Theory

The formulated model develops a relations between the average strain obtained during a stress-strain test, $\bar{\varepsilon}_i$, in which some average stress is sustained over a time interval, t_i, to the creep strain obtained during a creep test, ε_c. The average stress, $\bar{\sigma}$, sustained during the stress-strain test is equal to the creep stress, σ_c.

$$\bar{\sigma} = \sigma_c = \frac{\int_o^t \sigma(t)dt}{\int_0^t dt} \qquad [1]$$

The time interval, t, corresponds to the time over which the average stress, $\bar{\sigma}$, of the stress-strain test is sustained. Thus, creep strain is predicted from experimental stress-strain tests.

The starting point of the theory is based on a heuristic argument that for any two stress-strain curves, an exponential relation exists between the ratio of the strain rates and any two states of equal strain energy density. Consider two ramped tests at strain rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ and with equivalent strain energy densities given by $\sigma_1\varepsilon_1$ and $\sigma_2\varepsilon_2$ for the two curves, 1 and 2 respectively.[2]

$$\left(\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}\right)^m = \frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_2}{\varepsilon_1} \qquad [2]$$

This equation is used quite differently than in the reference. In this work, the exponent, m, is a fraction less than one and is a function of the product σε, m = m(σε). Rearranging equation [2], one is able to calculate values for m at equal strain energy density (SED) values between the two curves at $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$, respectively.

$$m = \frac{\log\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}{\log\left(\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}\right)} \qquad [3]$$

Once m is calculated from the experimental stress-strain data corresponding to $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$, a polynomial is fit to the log-log plot of m-SED.

A system of equations[3] is then solved simultaneously to determine 1) the predicted strain rate, $\dot{\varepsilon}_i$, corresponding to a chosen length of time, t_i, at which the creep strain is sought and 2) the time interval, t_r, over which $\bar{\sigma}$ is equal to the chosen σ_c for the reference stress-strain experiment at strain rate $\dot{\varepsilon}_r$.

Thus, the values of $\dot{\varepsilon}_r$, σ_c, and t_i, are all known. By substituting these knowns, the curve fit polynomial of log(m) as a function of log(σε), $m = 10^{\log(m)}$, the curve fit polynomial of stress as a function of time for the reference stress-strain experiment at strain rate $\dot{\varepsilon}_r$, and equation [6] into equations [4] and [5], the two unknowns are determinable for a chosen length of time and creep stress from the reference stress-strain experiment.

$$\int_0^{t_r} \sigma(t)dt - \sigma_c \left(\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_r} \right) t_i = 0 \qquad [4]$$

$$\left(\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_r} \right)^{\overline{m}(\overline{\sigma\varepsilon})} \left(\frac{\int_0^{t_r} \sigma(t)dt}{t_r} \right) - \sigma_c = 0 \qquad [5]$$

$$\overline{\sigma\varepsilon} = \frac{\sigma_c \dot{\varepsilon}_i t_i}{2} \qquad [6]$$

The creep strain ε_c and the creep exponent n_c are then calculated by substituting equations [7] and [8] into [9] and [10].

$$\bar{\varepsilon}_i = \frac{\dot{\varepsilon}_i t_i}{2} \qquad [7]$$

$$\bar{\varepsilon}_r = \frac{\dot{\varepsilon}_r t}{2} \qquad [8]$$

$$n_c = \frac{\log\left(\frac{\bar{\varepsilon}_i}{\bar{\varepsilon}_r} \right)}{\log\left(\frac{t_i}{t_r} \right)} \qquad [9]$$

$$\varepsilon_c = (\bar{\varepsilon}_i)(1 + n_c) \qquad [10]$$

Modeling the experimental creep strain through the relation $\varepsilon = kt^{n_c}$, in which k is a pre-exponential constant, the creep exponent in equation [9] is determined from the reference and predicted stress-strain curves. The average strain for the reference stress-strain curve, $\bar{\varepsilon}_r$, at $\dot{\varepsilon}_1 = \dot{\varepsilon}_r$ is determined by equation [7], in which case t is not chosen but corresponds to the length of time over which $\bar{\sigma} = \sigma_c$ and is determinable from equation [1]. Equation [1] is also used to determine t for $\dot{\varepsilon}_2$ with the corresponding curve fit polynomial of stress as a function of time.

To predict stress-strain values σ_i and ε_i at the predicted strain rate, $\dot{\varepsilon}_i$: $\dot{\varepsilon}_r$, $\dot{\varepsilon}_i$, and the stress and strain values from the reference stress-strain curve σ_r and ε_r and the corresponding value of m at each SED value determined from the two experimental stress-strain curves are substituted into equations [11] and [12].

$$\sigma_i = \sigma_r \left(\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_r} \right)^{m(\sigma\varepsilon)} \qquad [11]$$

$$\varepsilon_i = \varepsilon_r \Big/ \left(\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_r} \right)^{m(\sigma\varepsilon)} \qquad [12]$$

Results

The creep behavior was predicted for σ_c= 400 and 800 psi according to the Non-Linear Strain Energy Equivalence Theory for t_i = 60, 262,800, 525,600, 5,256,000, 10,512,000, and 26,280,000 minutes.

Results for $\dot{\varepsilon}_i$, t_i, t_r, n_c, and ε_c appear in Table 1 and Table 2 for σ_c= 400 and 800 psi, respectively. A plot of the predicted creep strain versus time is displayed in Figure 1 for both stress levels. The two experimental ($\dot{\varepsilon}_i$=0.03 and 0.0003 /minute) and the six predicted stress-strain curves for σ_c= 400 and 800 psi appear in Figures 2 and 3, respectively.

Discussion

The main principle of the theory is that one cannot simply extrapolate short-term creep strain out to long times due to the increasing creep exponent with time. The creep exponent varies with time as the creep mode, or the molecular response within the polymer, changes. Results from a 26-year creep experiment performed on virgin polyethylene by William Nichols Findley[4], validates this point. In Figure 4, a log-log plot of strain-time shows that as time progresses, the creep exponent increases from 0.0447 in region 1 (0 to 574 hours), to 0.0819 in region 2 (574-60,000 hours), and to 0.1084 in region 3 (60,000 to 230,500 hours).[5] We suggest that the creep mode, or material response, is changing within each region. In region 1 the creep mode is most likely due to deformation of the amorphous region. During region 2, it is hypothesized that the crystalline blocks line up next to each other. In region 3, the creep mode may be due to the “unzipping” of polymer chains from the crystalline blocks. As a consequence of the changing creep mode and increasing creep exponent, it was found that Findley’s experimental creep strain at 26 years was approximately 30 % greater than creep strain extrapolated from his short-term data out to 26 years.

The Non-Linear Strain Energy Equivalence theory follows this same trend. Notice the increase in n_c with increasing time in Tables 1 and 2. For σ_c=4 00 psi, n_c increases by 14.3 % from 0.0597 at t_i = 0.15 minutes to 0.0682 at 50 years, and for σ_c= 800 psi, n_c increases by 22.4 % from 0.0685 at t_i = 0.31 minutes to 0.0838 at 50 years. Thus, extrapolating creep behavior from short-term creep experiments is inadequate. The development of predictive techniques like the one utilized in this work is required to determine more accurate long-term creep behavior.

Conclusion

The Non-Linear Strain Energy Equivalence Theory predicts creep strain, the creep exponent, and stress-strain data at various strain rates from experimental compressive stress-strain data to longer times than the stress-strain experiments themselves.

While there is no existing data on long-term creep behavior of PS/HDPE immiscible polymer blend composites, the correlation of these findings with Findley's experimental results and results found in the earlier study of several plastic lumber formulations, suggests that a reasonable estimate of the creep strain is obtainable. Thus, the theory provides an alternative to conducting time-consuming, long-term creep tests while allowing much smaller safety factors, more efficient use of polymer blend materials with a high degree of confidence in structural loading applications, and an improvement in the cost competitiveness of these materials.

A computer software package was recently developed[6] to implement the Non-Linear Strain Energy Equivalence Theory, allowing the general user to predict the long-term creep behavior of a structural material for any structural loading application from short-term mechanical property measurements without having to grasp the theory itself. Thus, the lifetime of any material can be estimated quickly to avoid failure in structural loading applications subject to creep such as railroad crossties, I-beams, bridges, pallets, and structural panels.

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Table 1: Predicted creep behavior at σ_c = 400 psi for 35/65 % PS/HDPE.

$\dot{\varepsilon}_i$ (/minute)	t_i (Years)	t_i (Minutes)	t_r (Minutes)	n_c	ε_c
0.03		0.15		0.0597	0.0024
3.00E-04		20		Reference	0.0032
1.09E-04		60	21.08	0.0607	0.0035
4.26E-08	0.5	262800	27.87	0.0661	0.0060
2.24E-08	1	525600	28.57	0.0664	0.0063
2.63E-09	10	5256000	31.07	0.0675	0.0074
1.38E-09	20	10512000	31.88	0.0678	0.0077
5.89E-10	50	26280000	32.99	0.0682	0.0083

Table 2: Predicted creep behavior at σ_c = 800 psi for 35/65 % PS/HDPE.

$\dot{\varepsilon}_i$ (/minute)	t_i (Years)	t_i (Minutes)	t_r (Minutes)	n_c	ε_c
0.03		0.31		0.0685	0.0049
3.00E-04		44		Reference	0.0069
2.23E-04		60	44.17	0.0731	0.0072
9.69E-08	0.5	262800	61.82	0.0796	0.0137
5.14E-08	1	525600	63.71	0.0803	0.0146
6.32E-09	10	5256000	70.75	0.0830	0.0180
3.36E-09	20	10512000	73.16	0.0836	0.0191
1.45E-09	50	26280000	76.63	0.0838	0.0207

Figure 1: Predicted creep strain versus time at σ_c = 400 psi (■) and 800 psi (X) for 35/65 % PS/HDPE.

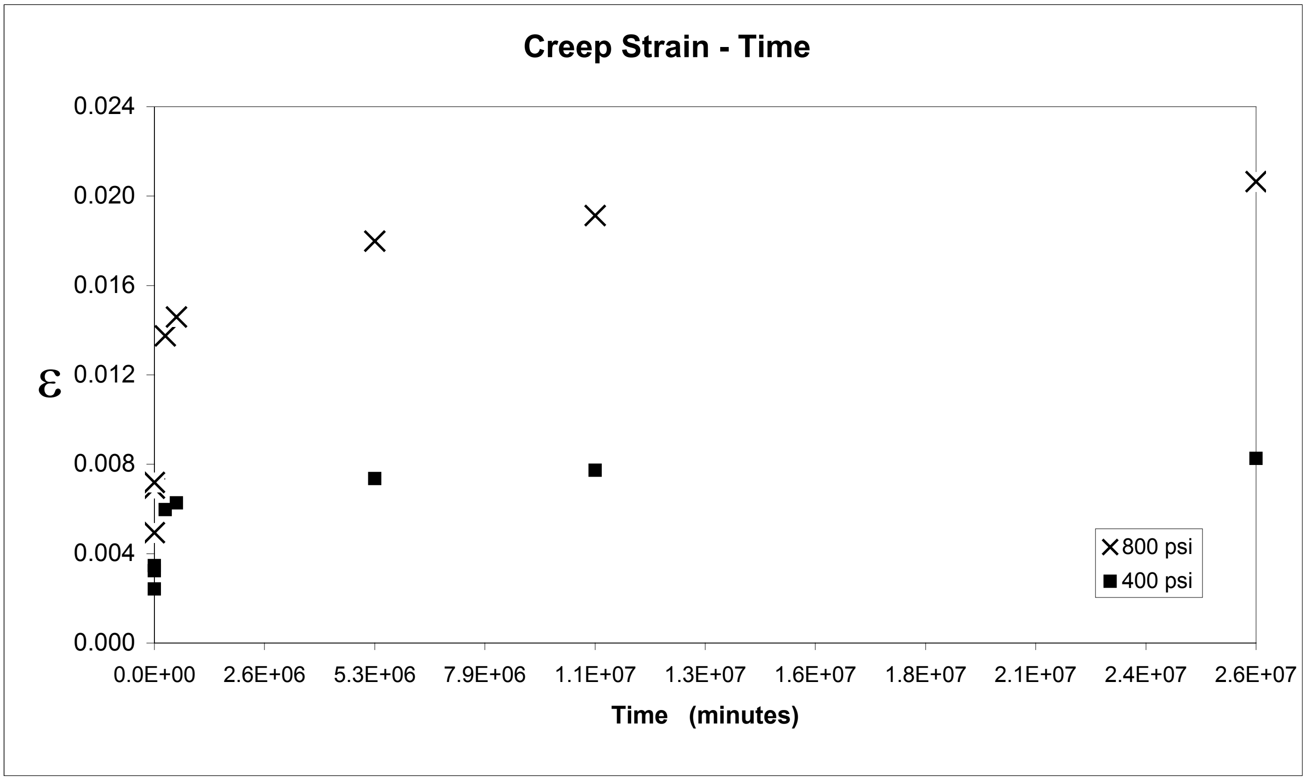


Figure 2: Predicted stress-strain curves for 35/65 % PS/HDPE from $\sigma_c = 400$ psi.

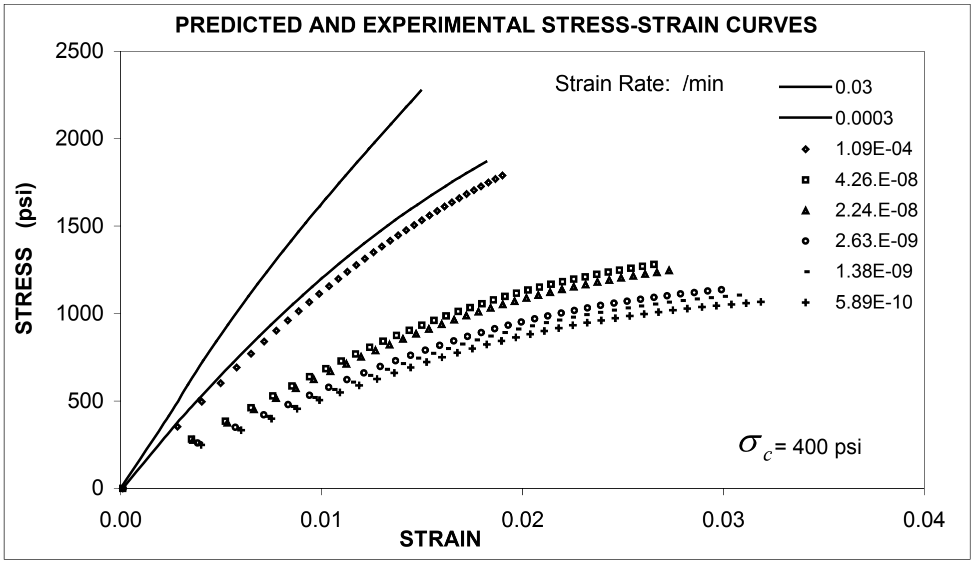


Figure 3: Predicted stress-strain curves for 35/65 % PS/HDPE from $\sigma_c = 800$ psi.

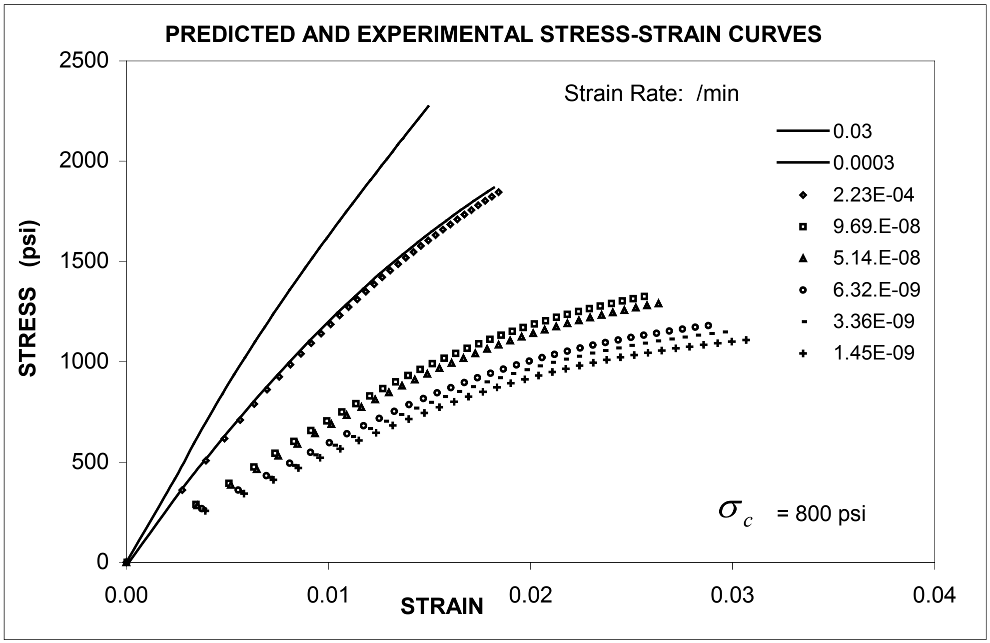
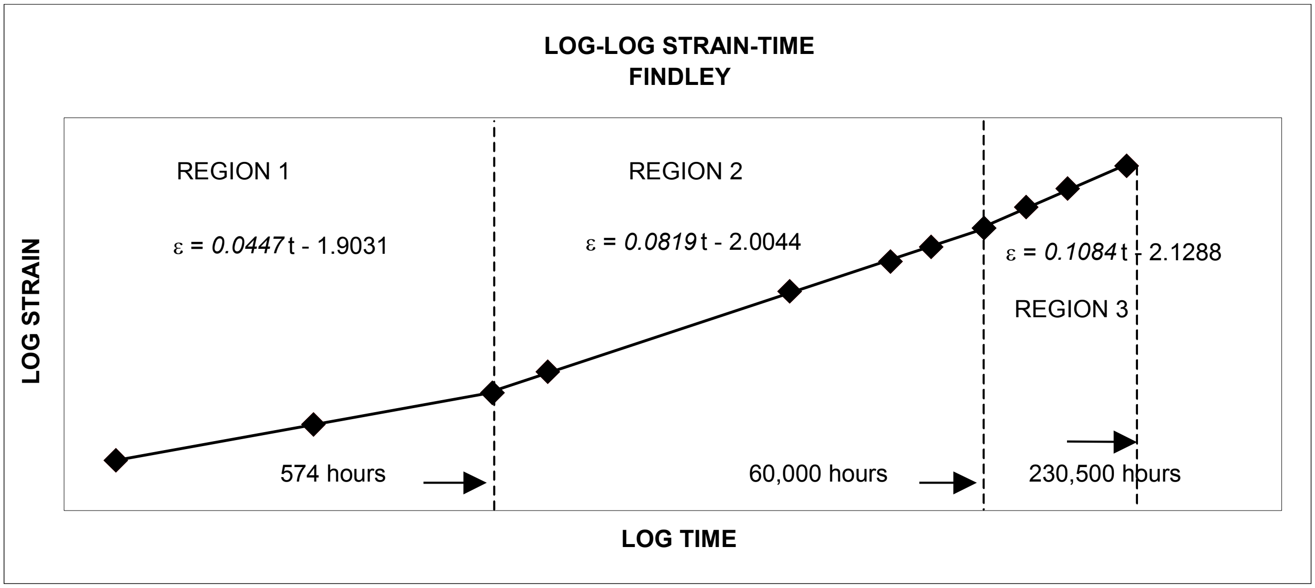


Figure 4: Log-log plot of Findley's 26-year creep strain versus time for virgin polyethylene.



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